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A CNPVaR-NPV Trade-off Model for Resource-Constrained Project Scheduling Problem Under Random Environment

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***Abstract*—Projects face various risks and opportunities throughout their lifecycle. Integrating risk management into the project progression is a crucial factor for project success and an essential component of project management. This paper aims to evaluate the trade-off between Conditional Net Present Value at Risk (CNPVaR) and Net Present Value (NPV) in a stochastic environment. By assuming the activity duration and cash flow as random variables and setting activity start and execution modes as decision variables, a CNPVaR-NPV trade-off model is established with the objectives of minimizing CNPVaR and maximizing NPV, under priority relationships, execution mode, resource supply, and deadlines constraints. Based on the characteristics of the model, an algorithm combining non-dominated sorting genetic algorithm II and Monte Carlo simulation is designed. The HVAC system project of Dongguan Jinyun Digital Park 5# Building is used as a case study to illustrate the effectiveness of the model and the algorithm. The results show that considering the CNPVaR-NPV trade-off model can effectively mitigate potential risks and reduce project losses.**

***Index Terms*—Multi-mode, resource constrained project scheduling problem, Conditional Net Present Value at Risk (CNPVaR) and Net Present Value (NPV), random environment**

I. INTRODUCTION

Resource Constrained Project Scheduling Problem (RCPSP) refers to determining the start times of each activity and order of each project activity under the constraints of resource supply and activity priority relationship [1–3]. RCPSP has been a widely studied issue in the past decades to minimize the project completion time [4, 5]. However, this objective may not be suitable for assessing project feasibility, as it ignores the economic aspects and risk situations, especially when extreme time and cost are likely [6]. For instance, many of the larger capital-intensive projects take a long time to run and involve substantial cash flow, small cash flow changes can significantly impact the profitability of the overall project. Therefore, scholars measure the feasibility of projects in terms of the time value of money and risk [7–9].

The research on the time value of money can be traced back to the 1970s, Russell [10] firstly proposed the time value of money in the project scheduling problem, known as Net Present Value (NPV), and a nonlinear programming model with deterministic cash flow was pretended to maximize NPV by advancing cash-in activities as early as possible and starting cash-out activities as late as possible Then, most scholars designed different models and algorithms to generate feasible baseline schedule plans to maximize project NPV [11–15]. The vast majority of the studies assume a static, deterministic environment with complete information where activity duration, resource requirements, and cost are known in advance [16]. It is well known that projects are susceptible to various types of disruptions in real life, such as resource interruptions, adverse weather conditions, delayed material supplies, changes in delivery dates, equipment failures, and necessary additional activities [17–20]. One or more activities may take longer than expected in the baseline schedule, and the schedule may not execute exactly as planned. Furthermore, changes in the baseline schedule will impact the NPV [21, 22]. Therefore, many scholars study the Excepted Net Present Value (ENPV) in RCPSP under random environment and construct different types of optimization models [12, 17, 22–24].

Among the traditional methods of tradeoff benefits and risks, value at risk (VaR) attracts a lot of scholars' attention [25]. VaR is a risk measure method used to determine the potential loss a portfolio may incur over a specific period [26]. VaR is a portfolio's maximum possible loss value under normal market conditions and at a certain confidence level. To better measure extreme risk situations and estimate potential risk more robustly, Rockafellar and Uryasev [27] introduced Conditional Value at Risk (CVaR) based on VaR. CVaR is the expected loss on an investment, provided the loss exceeds the VaR [28]. Dixit and Tiwari [29] formulated three portfolio selection and scheduling models: risk neutral, risk aversion, and portfolio compromise. A comparison of the results showed that the risk aversion model (CVaR model) ensured that the return in the worst case was as much as possible and could obtain a high return even at a low confidence level.

To comprehensively assess the financial feasibility and risk issues of a project throughout its life cycle, scholars applied the distribution function of NPV and the concept of VaR and proposed the concept of Net Present Value-at-

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Risk (NPVaR) [30, 31]. NPVaR focuses on the tail of the uncertain parameter distribution, which leads to faster optimization than considering all possible outcomes, and the resulting remains profitable even in the face of extremely adverse conditions [32]. Recently, Rezaei *et al*. [33] proposed the concept of Conditional Net Present Value at Risk (CNPVaR), which was an indicator that measures the risk of the NPV of an investment under certain conditions. It was based on the NPVaR but considered the impact of certain conditions. Assuming that the activity had multiple scenarios, the activity duration and cash flow were defined by a set of discrete alternatives with relative occurrence probabilities, and the optimization model in different scenarios was built to trade-off the CNPVaR-NPV.

However, Rezaei *et al*. [33] did not consider resource constraints. This paper describes the CNPVaR-NPV trade-off problem based on the literature of [33] and extends the value of the random variable to a more general case in the project. In addition, multiple execution modes of activities and resource constraints are considered. Considering the influence of the uncertain environment, the activity duration and cash flow are assumed as random variables, and the activity start time and execution mode are decision variables. Then, a CNPVaR-NPV trade-off model is constructed. Objective 1 is to minimize the CNPVaR, and objective 2 is to maximize the ENPV. A non-dominated sorting genetic algorithm II (NSGA-II), combined with a Monte Carlo simulation, has been designed. Finally, the HVAC system project of Dongguan Jinyun Digital Park 5# Building is used as a case study to verify the effectiveness of the model and algorithm.

The rest of this paper is organized as follows. Section II is the problem statement. Section III presents the model and its solution algorithm. Section IV presents the case background, model results, and discussion. Conclusions are drawn in Section V, and future research directions are proposed.

II. Problem Statement

Suppose the project is given by activity on node (AoN) representation. A project is treated as a directed set of arcs $G = (N, A)$, where the node set $N = (0, 1, \cdots, n+1)$ represents the set of activities, and $A: \{(i, j) \in A\}$ represents the set of arcs with priority relationship. Node 0 and node $n+1$ are virtual nodes that represent the start and end of the project, respectively, and their duration and resource usage are negligible [34]. This paper assumes the resource category limits to renewable resource (such as machines and workers), which can be used in cycles.

This paper discusses the CNPVaR and NPV trade-off in different modes to find a balance point. Different modes include different activity duration and cash flow. Assuming that the NPV is considered from the contractor's perspective. Negative cash flow or cash outflow includes cost related to resource, labor force, and equipment, while positive cash flow or cash inflow represents customer payments to contractors [35]. Cash flow is related to the performance of each activity, which is the sum of the contractor cost and the total amount client pays for the activity. The cash flows are discounted at the discount rate β after each activity.

1) Assumptions

1) Each activity has only a single mode of execution;
2) All predecessors must finish before the activity starts;
3) Each activity can start only the resource supply can fill its total demand;
4) All resource is only considered under the constraint of renewable resource;
5) No activity should start before the project starts or is in execution after the project has ended;
6) Payments are made regularly and final payments should be made when the project is completed;
7) The project must be completed before the deadline;
8) Activity duration and cash flow are random variables, and the mode and start time of the activity are decision variables.

2) Definitions and Parameters

To construct the model, Definition 1 and Definition 2 give the concepts of the VaR and CVaR, expressed in (1) and (2), respectively. The parameter implication of the CNPVaR-NPV trade-off model is also given in Table I.

TABLE I: THE PARAMETER IMPLICATION OF THE MODEL

Parameter	Implication
$\boldsymbol{\alpha}$	Confidence level
$\boldsymbol{C_i^m}$	Cash flow of activity i in mode m
$\boldsymbol{\beta}$	Discount rate per unit time
$\boldsymbol{s_i^m}$	Start time of activity i in mode m
$\boldsymbol{f_i^m}$	Completion time of activity i in mode m
$\boldsymbol{d_i^m}$	Duration time of activity i in mode m
$\boldsymbol{A_t}$	Set of ongoing activities at time t
$\boldsymbol{r_{ik}^m}$	The total demand account of resource k by activity i in mode m
$\boldsymbol{x_i^m}$	The activity i is executed in mode m
$\boldsymbol{R_k^m}$	The supply account of resource k in mode m
DD	Deadline

Definition 1 [36]. Assume X is a random variable and $\alpha \in (0, 1)$ is a confidence level, the VaR of X is the α-quantile, then

$$\mathrm{VaR}_X(\alpha) = \min\{x: F_X(x) \geq \alpha\} \tag{1}$$

Definition 2 [37]. Let X be a random variable with continuous distribution function, CVaR can be defined as the conditional expectation of X given that $X \geq \mathrm{VaR}_X(\alpha)$,

$$\mathrm{CVaR}_X(\alpha) = E[X|X \geq \mathrm{VaR}_X(\alpha)] = \frac{1}{(1-\alpha)} \int_{\mathrm{VaR}_X(\alpha)}^{+\infty} x dF_X(x) \tag{2}$$

where $F_X(x)$ is the cumulative distribution function of X.

III. CNPVaR-NPV Trade-off Model

Accurately assessing risk allows managers to adjust status in time to avoid loss and to better maximize project profit. According to the definitions of VaR and CVaR, the expression of CNPVaR is presented in (3a) [33]:

$$\mathrm{CNPVaR} = E[\mathrm{NPV}|\mathrm{NPV} \geq \mathrm{NPVaR}(\alpha)] \tag{3a}$$

Different from traditional literature [33], this paper generalizes random variables as continuous random variables. Therefore, CNPVaR needs to be calculated through the integral as shown in the following equation, i.e., (3a) can be written as (3b):

$$\text{CNPVaR} = \frac{1}{1-\alpha}\int_{\text{NPVaR}(\alpha)}^{+\infty} x dF_{\text{NPV}}(x). \quad (3b)$$

The economic objective must be taken into consideration in order to measure the feasibility of the project. Scholars often use NPV as the economic objective function to evaluate future profit better [38]. When NPV is greater than 0, the project can be invested. On the contrary, if the NPV is less than 0, the project will lose money and be unsuitable for investment. The NPV can be calculated as the sum of the cash flow product and the discount rate for all activities:

$$E[\text{NPV}] = E\left[\sum_{i\in V} C_i^m \cdot \delta^{s_i^m}\right] \quad (4)$$

The cash flow of the activity is assumed to be the random variable, thus the expected NPV needs to be considered.

This paper considers the end-start precedence relationship between activities. For $\forall(i,j) \in A$, no matter what the mode is, the start time of the activity j must be after the completion of the preceding activity i, expressed as

$$s_j^m \geq s_i^m + d_i^m \quad \forall(i,j) \in A,\ \forall m \in M \quad (5)$$

Every project must have a completion time limit because time and cost are two relatively restrictive objectives. The manager must pay a specific cost if wants a short construction period. Therefore, in the constraint, the project deadline must be considered. In mode m, the last activity n+1 completion time cannot exceed the specified project deadline (DD) specified by the project. The expression is given in Equation (6).

$$f_{n+1}^m \leq \text{DD} \quad (6)$$

Resource constraint is an indispensable item. Resource constraint means that the resource usage must not exceed the total resource supply at any time and in any mode. The amount for resource k usage in execution mode m for all ongoing activities cannot exceed the total resource supply R_k^m. The resource constraint is shown in (7):

$$\sum_{i\in A_t} r_{ik}^m \leq R_k^m \ \forall m \in M \quad (7)$$

where A_t represents the set of ongoing activities at time t.

Different execution modes correspond to different activity duration and cash flow. The activity can choose the appropriate support mode according to the project objective but with only one execution mode. The expression of the execution mode is given as

$$\sum_{m\in M} x_i^m = 1 \quad (8)$$

and

$$\begin{cases} x_i^m = 0, \text{if the activity } i \text{ does not choose the mode } m, \\ x_i^m = 1, \text{if the activity } i \text{ chooses the mode } m. \end{cases}$$

A. Model Building

Before the project is implemented, the impact of risk on the project must be considered to make a correct assessment. A correct understanding of risk will help managers make correct decisions, timely strategy adjustments, and better realize the objective of maximizing profit. Suitable scheduling of activities to minimize loss is necessary for risk-averse managers. Therefore, this model wants to find a trade-off schedule plan between minimizing the CNPVaR and maximizing the NPV under priority relationship, project deadline, resource supply and execution mode constraints. Assuming that the activity duration and cash flow are random variables, the CNPVaR-NPV trade-off model can be obtained.

$$\begin{cases} \min \text{CNPVaR} \\ \max E[\text{NPV}] \\ s.t. \\ s_j^m \geq s_i^m + d_i^m \quad \forall(i,j) \in A, \qquad \forall m \in M \\ f_{n+1}^m \leq \text{DD} \\ \sum_{i\in A_t} r_{ik}^m \leq R_k^m \\ \sum_{m\in M} x_i^m = 1 \\ \text{CNPVaR} = \dfrac{1}{1-\alpha}\displaystyle\int_{\text{NPVaR}(\alpha)}^{+\infty} x dF_{\text{NPV}}(x) \\ E[\text{NPV}] = E\left[\sum_{i\in V} C_i^m \cdot \delta^{s_i^m}\right] \end{cases}$$

The objective 1 of the CNPVaR-NPV trade-off model is to minimize CNPVaR, the objective 2 is to maximize NPV, the constraint 1 is the precedence relationship constraint of the activity, the constraint 2 is the deadline constraint of the project, the constraint 3 is the resource supply constraint, and the constraint 4 is the constraint of the activity execution mode. Managers need to find a scheduling plan that can meet the objectives and constraints of the model.

B. Algorithm Design

TABLE II: THE SOLUTION ALGORITHM STEPS OF THE MODEL

Start	
1:	Determine the algorithm parameters which includes: pop (population quantity), gen (evolutional generation), P_c (crossover probability), P_m (mutation probability).
2:	Import case data, including the number of activities, the cost, duration, resource usage, and resource supply under different execution modes.
3:	The adjacency matrix is used to import the close preceding activity relationship of each activity.
4:	Create an initial population.
5:	Calculate the fitness value.
6:	Close judgment of previous activity relationship;
7:	Beyond the deadline judgment;
8:	**For** i=1: Number of activities
9:	Simulate project values based on input parameters.
10:	Calculate the cash flow in each mode.
11:	Generate NPV distribution.
12:	**End for**
13:	Calculate CNPVaR.
14:	Calculate E(NPV).
15:	Determine the constraints of resource supply.
16:	Evaluate the initial solution.
17:	Perform fast non-dominated rank of initial solutions.
18:	Calculate crowding distances (CDs) for sorting.
19:	Sort populations according to rank and CDs.
20:	**For** j = 1: gen **do**:
21:	Generate parent 1 $O1_j$;
22:	Generate parent 2 $O2_j$;
23:	Generate new population O_j by crossover and mutation.
24:	Sort O_j by non-dominated sorting and CDs.
25:	Generate child population O_{j+1} as size as pop.
26:	**End for**
27:	Stop criterion: The final number of iterations when the algorithm stops and the final solution is obtained.

According to the characteristics of the model, an algorithm integrating NSGA-II and Monte Carlo simulation is designed, and MATLAB R2016 software is used to solve the model. The algorithm steps are given in Table II.

IV. Case Analysis

This section uses the data center project as a case study to verify the effectiveness of the model and algorithm. Data centers have the characteristics of high investment and complex construction, and the project return and risk are relatively high. The selected project center project is the the HVAC system project of Dongguan Jinyun Digital Park 5# Building.

A. Background and Data

Dongguan Jinyun Digital Park 5# Building is located in Dongguan City, Guangdong Province, China. A 3-storey building with a height of 16.15 meters, mainly serving financial securities service companies and large internet companies. The first floor is 6.6 meters high, and the second floor is 5.1 meters high, the three-story height is 4.2 meters, and the use function is a data center (C-level workshop) with a total construction area of 9,626 square meters. Dongguan Jinyun Digital Park 5# project mainly includes HVAC, electromechanical, and decoration systems. HVAC system is more complex and will face more problems, so this paper chooses HVAC system as the research case.

Through investigation and research, the HVAC system mainly includes 18 activities. Assuming that each activity duration and cash flow are random variables. According to the project parameter interval data provided by the project manager, we get the probability distributions that are shown in Table III and Table IV. This paper divides the mode into rush mode and normal mode. Table III shows the parameters of each activity in the rush mode, and Table IV shows the parameters of each activity in the normal mode. Resource 1 is a common work type with supply of 60 persons, and resource 2 is a technical work type with supply of 33 persons. The deadline is set at 95 days. Assume the β is 0.95, and the value of α is 0.7.

TABLE III: Related Parameters in Rush Mode

Node	Activity	Priority relationship	r_{i1}^1	r_{i2}^1	d_i^1	C_i^1
0	/	/	/	/	/	/
1	Cooling tower construction	/	120	90	$U(14,16)$	$U(-10,-15)$
2	Cold storage tank construction	/	70	30	$U(9,11)$	$U(-6,-8)$
3	Freezer station pick up	/	220	198	$U(21,23)$	$U(-14,-15)$
4	Precision air conditioning in place	/	136	0	$U(16,18)$	$U(15,17)$
5	Precision air conditioner pick-up	4	252	252	$U(27,29)$	$U(10,11)$
6	Freezing station suppression	3	25	0	$U(4,6)$	$U(-8,-9)$
7	Precision air conditioner pick-up and press	/	125	125	$U(24,26)$	$U(-8,-10)$
8	The fan coil unit is in place and connected to the machine	/	324	180	$U(35,37)$	$U(7,10)$
9	Duct construction	/	380	0	$U(37,39)$	$U(-4,-5)$
10	Pipe installing	/	319	290	$U(28,30)$	$U(10,11)$
11	Air conditioner pick-up	/	266	304	$U(37,39)$	$U(-8,-10)$
12	Pipe flushing	/	33	0	$U(10,12)$	$U(7,10)$
13	Diesel steel platform installation	/	100	60	$U(9,11)$	$U(9,14)$
14	Air-conditioning room return air duct insulation	/	99	0	$U(10,12)$	$U(-9,-11)$
15	Corridor horizontal pipe insulation	/	220	0	$U(21,23)$	$U(20,22)$
16	Insulation construction	/	243	0	$U(26,28)$	$U(-10,-11)$
17	Color shell, aluminum skin construction	/	170	0	$U(16,18)$	$U(17,19)$
18	/	/	/	/	/	/

TABLE IV: Related Parameters in Normal Mode

Node	Activity	Priority relationship	r_{i1}^2	r_{i2}^2	d_i^2	C_i^2
0	/	/	/	/	/	/
1	Cooling tower construction	/	120	80	$U(19,21)$	$U(-8,-10)$
2	Cold storage tank construction	/	56	28	$U(13,15)$	$U(-4,-5)$
3	Freezer station pick up	/	216	189	$U(26,28)$	$U(-11,-13)$
4	Precision air conditioning in place	/	150	0	$U(24,26)$	$U(18,20)$
5	Precision air conditioner pick-up	4	245	245	$U(34,36)$	$U(13,15)$
6	Freezing station suppression	3	28	0	$U(6,8)$	$U(-5,-7)$
7	Precision air conditioner pick-up and press	/	120	90	$U(29,31)$	$U(-6,-8)$
8	The fan coil unit is in place and connected to the machine	/	240	120	$U(39,41)$	$U(8,10)$
9	Duct construction	/	282	0	$U(46,48)$	$U(-2,-3)$
10	Pipe installing	/	252	216	$U(35,37)$	$U(13,15)$
11	Air conditioner pick-up	/	180	216	$U(35,37)$	$U(-7,-8)$
12	Pipe flushing	/	30	0	$U(14,16)$	$U(6,10)$
13	Diesel steel platform installation	/	112	70	$U(13,15)$	$U(10,11)$
14	Air-conditioning room return air duct insulation	/	96	0	$U(15,17)$	$U(-7,-9)$
15	Corridor horizontal pipe insulation	/	175	0	$U(24,26)$	$U(22,24)$
16	Insulation construction	/	150	0	$U(29,31)$	$U(-9,-10)$

Node	Activity	Priority relationship	r_{i1}^2	r_{i2}^2	d_i^2	C_i^2
17	Color shell, aluminum skin construction	/	114	0	$U\,(18,20)$	$U(19,22)$
18	/	/	/	/	/	/

B. Results and Discussion

According to the case data and the convergence rate of the optimal solution, the core parameters of the NSGA-II in this paper are determined as follows: parameter population size pop=800 evolution algebra gen=100 mutation probability p_m=0.9, crossover probability p_c=0.9. The optimal solutions are obtained in Fig. 1. The solution results of the model are shown in Table V and Table VI, in which the different schedule strategies are compared. When the CNPVaR is 126,279 yuan, the ENPV is 113,042 yuan under the activity finish time schedule (0, 23, 1, 13, 44, 47, 39, 45, 8, 2, 49, 1, 40, 7, 32, 7, 1, 19, 83) and execution mode (0, 1, 1, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 2, 1, 2, 1, 2, 0); when the CNPVaR is 96,293 yuan, the ENPV is 85,399 yuan under the activity finish time schedule (0, 11, 21, 41, 29, 35, 29, 42, 11, 29, 1, 46, 1, 12, 27, 1, 42, 1, 82) and execution mode (0, 2, 2, 1, 1, 1, 1, 1, 2, 2, 1, 2, 2, 2, 1, 1, 2, 2, 0).

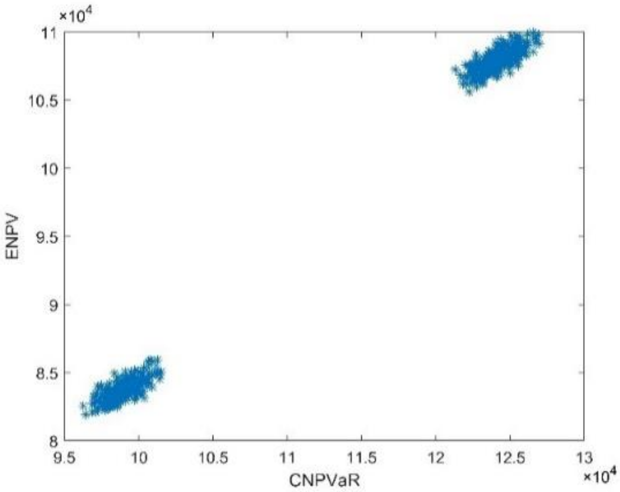

Fig. 1. Optimal solution of model.

TABLE V: OBJECTIVE FUNCTION'S RESULT (I) OF MODEL

Objective function	Value	Schedule
CNPVaR	126,279 yuan	(0, 23, 1, 13, 44, 47, 39, 45, 8, 2, 49, 1, 40, 7, 32, 7, 1, 19, 83)
ENPV	113,042 yuan	

TABLE VI: OBJECTIVE FUNCTION'S RESULT (II) OF MODEL

Objective function	Value	Schedule
CNPVaR	96,293 yuan	(0, 11, 21, 41, 29, 35, 29, 42, 11, 29, 1, 46, 1, 12, 27, 1, 42, 1, 82)
ENPV	85,399 yuan	

From these two pairs of results, managers can see that the higher ENPV, the higher CNPVaR. In other words, the higher the risk when trying to gain more profit. Managers can evaluate the project's ability to withstand risk in order to choose the appropriate optimization option. If the manager is a risk-averse person, then solution 2 (Table VI) is more suitable than solution 1 (Table V). If the manager wants to gain more profit and has a higher level of stress tolerance, then solution 1 (Table V) is a very suitable option with ENPV 3 units higher than solution 2 (Table VI), and CNPVaR less than 3 units higher than solution 2 (Table VI). The project completion time from optimization solution 1 is 83 days, and the project completion time from optimization solution 2 is 82 days. The difference between the above two solutions is not significant from view of the project completion time, so the choice is made on the basis of risk and profit.

C. Sensitivity Analysis

The relationship between ENPV and CNPVaR is shown in Fig. 2. Fig. 2 reveals that NPV increases with the increase of CNPVaR and decreases with the decrease of CNPVaR. The above results show that ENPV and CNPVaR have an overall linear relationship. When a project's risk (CNPVaR) increases, investment uncertainty increases, so a higher expected profit (NPV) is required to offset the risk from uncertainty. Conversely, when the CNPVaR of a project decreases, so does the uncertainty of the investment, and therefore the NPV decreases. Local dispersion is due to the randomness of the project parameters.

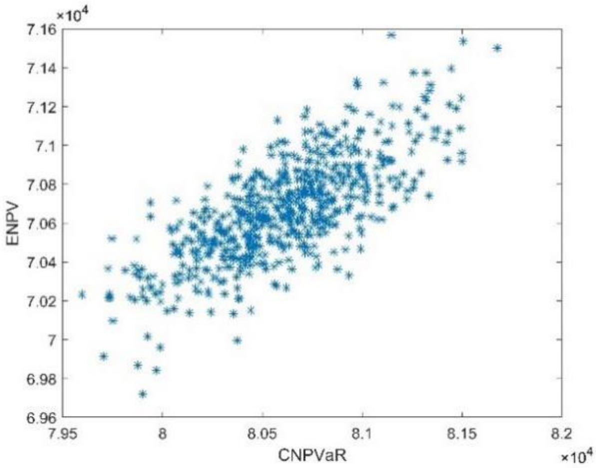

Fig. 2. The relationship between CNPVaR and ENPV.

The effect of resource supply change on completion time as shown in Fig. 3. Changes in resource 1 supply have a more pronounced impact on project completion time than resource 2 supply. Given that resource 1 supply is significantly more available and utilized than resource 2 supply, managers may need to prioritize monitoring the supply of resource 1 and prepare to adjust the project plan and resource allocation based on the supply of resource 1 and resource 2 changes. The proactive approach is necessary to avoid any undue impact on the project schedule, particularly when the availability of resource 1 supply decreases. By paying closer attention to the availability of resource 1 supply and taking pre-emptive measures, managers can mitigate potential disruptions and ensure smoother project execution.

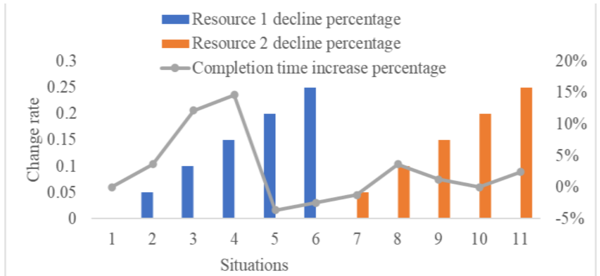

Fig. 3. The effect of resource supply change on completion time of the model.

D. Comparison CNPVaR-NPV Trade-off Model and Traditional Model

This paper uses Rezaei *et al*'s model [33] as the traditional model. The objectives of the traditional model are the same as those of CNPVaR-NPV trade-off model. The difference is that the parameters of the traditional model are discrete random variables in which the duration and cash flow of each activity are expressed using scenario analysis. Each scenario has the fixed activity duration and cash flow value. For the traditional model, two optimal solutions are obtained, as shown in Fig. 4 and Table VII.

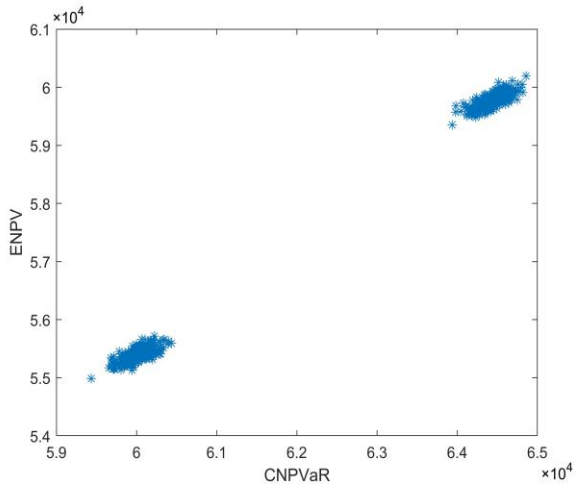

Fig. 4. Optimal solutions of the Rezaei's [33] literature model.

TABLE VII: OPTIMAL SOLUTION OF THE REZAEI *ET AL'S* [33] MODEL

Solution	Objectives	Value
Solution 1	CNPVaR	64,352 yuan
	ENPV	59,877 yuan
Solution 2	CNPVaR	62,896 yuan
	ENPV	55,419 yuan

The results of the traditional model are slightly different from those of CNPVaR-NPV trade-off model. The traditional model is the random variable in discrete situations, while CNPVaR-NPV trade-off model generalizes discrete variables to continuous variables. Although there are differences in the theoretical calculation methods of CNPVaR and ENPV, during the case application, it is found that the difference in results between the two objective functions is negligible. This difference is mainly due to two reasons. First, computer-aided algorithms are applied in the model solution process. The algorithm uses the discrete approximation method to solve the optimization problem of continuous random variables. Although this method is more accurate than the original discrete case, it still cannot keep up with the theoretical analytical solution. Secondly, in the case of this paper, the probability distribution of the assumed continuous random variable is uniformly distributed, and the difference between the upper and lower bound parameters is small, which results in a slight difference.

From the comparison results of Rezaei *et al*'s [33] traditional model and CNPVaR-NPV trade-off model, the following conclusions can be obtained.

Closer to the actual situation. CNPVaR-NPV trade-off model extends the traditional model from discrete situations to continuous situations, which is more in line with the continuity characteristics of resources and activity duration in actual projects, therefore, closer to the actual situation.

Higher calculation accuracy. Although CNPVaR-NPV trade-off model is different from the traditional model in theoretical calculation methods, the calculation accuracy of optimization problems with continuous random variables is higher in practical applications.

Consider more factors. CNPVaR-NPV trade-off model consider the situation of continuous random variables and considers more factors than the traditional model, making the scheduling plan more comprehensive and detailed.

V. CONCLUSION

This paper considers the CNPVaR-NPV trade-off problem in random environments, proposes a multi-mode optimization model to minimize CNPVaR and maximize NPV. First, this paper assumed the activity duration and cash flow were random variables, and the activity start time and the mode were decision variables. Then, a CNPVaR-NPV trade-off model was built under priority relationship, due date, resource supply and activity execution mode constraints. The NSGA-II algorithm combined with Monte Carlo simulation was designed to solve this model. Finally, the effectiveness of the model and algorithm was verified by taking the HVAC system project of Dongguan Jinyun Digital Park 5# Building as the case study. The results showed that considering the balance between CNPVaR and NPV can help risk-averse managers gain a comprehensive understanding of the potential value and risks of the project and can help project managers prepare for various possible outcomes, ensure that risk is minimized while maximizing profit. However, this paper only considered CNPVaR-NPV trade-off problem from the perspective of the risk-averse managers. In future research, the attitudes of different types of project managers toward risk to study the problem can be considered.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Qian Zhang and Sin Yin Teh performed the modeling; and analyzed the data; Qian Zhang, Sin Yin Teh and Cheang Peck Yeng Sharon wrote the paper and designed algorithm; Sin Yin Teh and Cheang Peck Yeng Sharon revised the paper.

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